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THE OPTIMUM AIRFOILS AT LOW ANGLES OF ATTACK IN A SUPERSONIC GAS FLOW†

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The problem of determining the plane airfoil which has the minimum wave drag in a uniform supersonic free stream is considered. The length, thickness, angle of attack, lift force and moment of the airfoil with respect to the origin of coordinates are assumed to be given. The consideration is confined to the class of bodies for which attached shock waves occur. It is assumed that the flow is supersonic and there are no internal shock waves in the regions where the flow is affected by the components of the required contour. Numerical results are given.

The following variational problem was considered in a previous paper [1]: of all the airfoils of specified thickness in a uniform supersonic free stream with a specified angle of attack it is required to obtain the one which has the minimum wave drag. The solution of the problem was sought in the class of airfoils for which attached shock waves occur, while the flow is supersonic and there are no internal shock waves in the regions in which the flow is affected by the generatrices of the airfoil. The necessary conditions for an extremum were derived and calculations were carried out for airfoils having two negative sharp bends and angles of attack for which Prandtl–Mayer flow occurs in the neighbourhood of the leading sharp edge from the windward side.

The difference between the present paper and [1] is as follows. First, we consider here the case of low angles of attack, i.e. those for which in the neighbourhood of the leading sharp edge attached shock waves occur both from the windward and leeward edges. Second, we take into account, as additional isoperimetric conditions, the lift force and the moment of the airfoil with respect to the origin of coordinates. Third, the necessary conditions for the functional to have an extremum are written for an airfoil with newly introduced negative sharp bends at the points E_1 and E_2 (Fig. 1) (in [1] sharp bends were only assumed at the points D_1 and D_2). The introduction of additional sharp bends in the airfoil is important for the new formulation of the problem. As numerical calculations showed, it is impossible to satisfy all the necessary conditions for an extremum on an airfoil which has sharp bends only at the points D_1 and D_2. A consideration of this problem in model form [1] also leads to this conclusion.

1. FORMULATION OF THE PROBLEM

We consider the problem of determining the plane airfoil $AD_1E_1BE_2D_2A$ (Fig. 1) having minimum wave drag in a supersonic flow of a perfect gas. We will assume that in the regions R_1 and R_2 where the flow and the airfoil influence one another the flow is supersonic and there are no internal shock waves, that the generated head shock waves AC_1 and AC_2 are attached and that the specified maximum thickness of the airfoil T, measured in a direction perpendicular to the chord AB (the length of the section E_1E_2), is reached at a single (but not definite) position of the points E_1 and E_2. In addition, the required airfoil must have a specified length and specified angle of attack and must, in general, have specified values of the lift force and moment about the point A. (Some of these constraints may be ignored when solving specific problems.)

In Fig. 1 (in addition to the lines mentioned) mn is a streamline and all the remaining lines are characteristics of the first or second families. The free stream is parallel to the x axis. We will assume that the origin of Cartesian coordinates (x, y) coincides with the point A.

The following notation is used: u and v are the projections of the velocity vector onto the x and y axes, respectively, referred to the critical flow velocity a_*, ρ is the gas density referred to the gas density in the free stream ρ_∞, p is the pressure, referred to $\rho_\infty a_*^2$, $\varkappa$ is the adiabatic index, α is the Mach angle, θ is the angle between the velocity vector and the x axis, γ is the angle of attack (the angle between the

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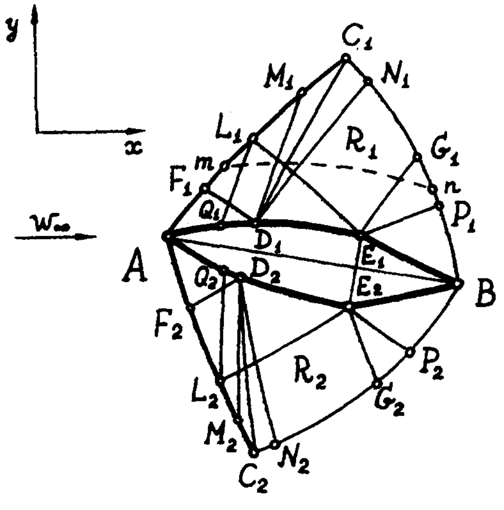

Fig. 1.

velocity vector of the unperturbed flow and the chord of the airfoil AB), φ is the entropy function and ψ is the stream function, introduced in the usual way by the equation

$$d\psi = \rho u dy - \rho v dx$$

where the value of ψ will be assumed to be zero on the airfoil. The problem will be analysed in (x, ψ) variables.

The steady non-isentropic flow of a perfect gas in the regions R_1 and R_2 is described by the following system of equations (the first of these is the equation of continuity and the second is the equation of conservation of momentum projected onto the y axis)

$$L_1 = \frac{\partial}{\partial x}\left(\frac{1}{\rho u}\right) - \frac{\partial}{\partial \psi}\left(\frac{v}{u}\right) = 0, \quad L_2 = \frac{\partial v}{\partial x} + \frac{\partial p}{\partial \psi} = 0$$
$$\frac{u^2+v^2}{2} + \frac{\varkappa p}{(\varkappa-1)\rho} = \frac{\varkappa+1}{2(\varkappa-1)}, \quad p = \rho^{\varkappa}\varphi^{(\varkappa-1)}(\psi) \tag{1.1}$$

The wave drag, the lift force and the moment of the airfoil with respect to the origin of coordinates are given by the following relations

$$X = \int_{x(A)}^{x(B)} \{p[x,\eta_2(x)]\eta_2'(x) + p[x,\eta_1(x)]\eta_1'(x)\}dx, \; Y = \int_{x(A)}^{x(B)} \{p[x,\eta_2(x)] - p[x,\eta_1(x)]\}dx$$
$$M = \int_{x(A)}^{x(B)} \{p[x,\eta_2(x)][x+\eta_2(x)\eta_2'(x)] - p[x,\eta_1(x)][x+\eta_1(x)\eta_1'(x)]\}dx \tag{1.2}$$

Here $\eta_1(x)$ and $\eta_2(x)$ are functions describing the components AD_1E_1B and AD_2E_2B of the airfoil in the (x, y) system of coordinates (here we assume that the lower part of the airfoil is reflected symmetrically in the x axis).

Since the coordinates of the point B are specified (Fig. 1) (for fixed coordinates of the point A, specifying the coordinates of the point B is equivalent to specifying the length of the airfoil $|AB|$ and the angle of attack γ), the required functions $\eta_1(x)$ and $\eta_2(x)$ must satisfy the isoperimetric conditions

$$r_1 = \int_{x(A)}^{x(B)} \eta_1'(x)dx - y(B) = 0, \quad r_2 = \int_{x(A)}^{x(B)} \eta_2'(x)dx + y(B) = 0 \tag{1.3}$$

By specifying the length of the section E_1E_2 (the maximum thickness of the airfoil T) we obtain the isoperimetric condition and the final relation

$$r_3 = \int_{x(A)}^{x(E_1)} \eta_1'(x)dx + \int_{x(A)}^{x(E_2)} \eta_2'(x)dx = T\cos\gamma, \quad r_4 = x(E_1) - x(E_2) = T\sin\gamma \tag{1.4}$$

The following impermeability conditions hold along the components of the airfoil AD_1E_1B and AD_2E_2B

$$v[x, \eta_i(x)] / u[x, \eta_i(x)] - \eta_i'(x) = 0, \quad i = 1, 2 \tag{1.5}$$

while along the lines AC_i the following relations for the parameters of the flow in front of the shockwave and behind it hold

$$K_1^{(i)} = \frac{v}{u}x_i' + \frac{1}{\rho u} - \frac{1}{w_\infty} = 0, \quad K_2^{(i)} = \frac{p - p_\infty}{w_\infty} + u - w_\infty = 0, \quad K_3^{(i)} = (p_\infty - p)x_i' + v = 0 \tag{1.6}$$

Relations (1.6) are the conditions on the discontinuities for the gas-dynamics equations written in (x, ψ) variables. The first of these corresponds to the equation of continuity while the second and third are the equation of conservation of momentum projected onto the x and y axes, respectively. Here $w = \sqrt{(u^2 + v^2)}$ is the modulus of the velocity, the subscript ∞ denotes free-stream quantities and $x_i(\psi)$ defines the line AC_i in the (x, ψ) plane.

The variational problem is formulated as follows: for a specified free-stream it is required to find the functions $\eta_1(x)$ and $\eta_2(x)$ which satisfy conditions (1.3) and (1.4) and which minimize the first functional (1.2) for given values of the second and third functionals (1.2), when the differential relations (1.5) are satisfied on the lines AD_iE_iB, relations (1.6) on the lines AC_i, and relations (1.1) in the regions R_i ($i = 1, 2$).

2. THE NECESSARY CONDITIONS FOR OPTIMALITY

Denoting the constant multipliers by σ_0, μ_0, μ_1, μ_2, λ and the variable multipliers by ε_i, σ_{il}, h_1, h_2, we can write the Lagrange functional as follows:

$$\begin{aligned} I &= X + \sigma_0 Y + \mu_0 M + \mu_1 r_1 + \mu_2 r_2 + \lambda r_3 + \\ &+ \sum_{i=1}^{2} \left\{ \int_{x(A)}^{x(B)} \varepsilon_i(x)\left[\eta_i' - \frac{v}{u}\right]dx + \int_{\psi(A)}^{\psi(C_i)} \sum_{l=1}^{3} \sigma_{il}(\psi)K_l^{(i)}d\psi \right\} + \\ &+ \iint_{R_1 \cap R_2} [h_1(x,\psi)L_1 + h_2(x,\psi)L_2]dxd\psi \end{aligned}$$

Here we have assumed a discontinuity in the multipliers h_1 and h_2 along the characteristics E_iL_i, C_iD_i and F_iD_i.

Using the method developed in [2–7], we can obtain the first variation of the functional I, and by equating this to zero we obtain the necessary conditions for an extremum. These conditions have the following form.

In the regions of influence R_1 and R_2 the variable Lagrange multipliers h_1 and h_2 are found from the following system of partial differential equations

$$\frac{a^2 - u^2}{\rho u^2 a^2}\frac{\partial h_1}{\partial x} - \frac{v}{u^2}\frac{\partial h_1}{\partial \psi} + \rho u \frac{\partial h_2}{\partial \psi} = 0, \quad \frac{v}{\rho u a^2}\frac{\partial h_1}{\partial x} - \frac{1}{u}\frac{\partial h_1}{\partial \psi} + \frac{\partial h_2}{\partial x} - \rho v \frac{\partial h_2}{\partial \psi} = 0 \tag{2.1}$$

where $a = \sqrt{(\varkappa p/\rho)}$ is the velocity of sound. When $w > a$, this system is hyperbolic, has two families of characteristics, the directions of which coincide with the characteristic directions of system (1.1) and the compatibility conditions along which are represented in the form

$$dh_1 \mp g^2 dh_2 = 0, \quad g = \sqrt{\rho u^2 \operatorname{tg}\alpha} \tag{2.2}$$

(Here and everywhere below the upper sign corresponds to the characteristics of the first family while the lower sign corresponds to the characteristics of the second family.)

The multipliers h_1 and h_2 must satisfy the following conditions on the airfoil

$$\frac{\partial h_1}{\partial x}(x,0) = -\frac{\partial p}{\partial x}[1+(-1)^i \mu_0 \eta_i], \quad h_2(x,0) = \frac{v}{u} + (-1)^i \left[\sigma_0 + \mu_0\left(x + \eta_i \frac{v}{u}\right)\right] \tag{2.3}$$

The following condition must be satisfied on the closing characteristics C_1B and C_2B

$$h_1 - g^2 h_2 = 0 \tag{2.4}$$

while on the characteristics along which discontinuities of the multipliers h_1 and h_2 are permitted, the values of the discontinuities are related by the equations

$$\Delta h_1 \pm g^2 \Delta h_2 = 0 \tag{2.5}$$

The following equation must be satisfied along each streamline

$$E_*^i(\psi) = \left[\frac{W_i(\psi)}{\rho v(1-p_\infty/p)}\right]_m + \int_{x_m}^{x_n} \left[\frac{\cos^2\alpha - \varkappa}{\varkappa \rho u}\frac{dh_1}{dx} + \frac{v\sin^2\alpha}{\varkappa}\frac{dh_2}{dx}\right]dx +$$

$$+\left[\frac{h_1}{\rho u} - p h_2 \frac{\cos(\theta-\alpha)}{\rho a}\right]_n + \sum_{S_j}\left[\frac{\Delta h_1}{\rho u} + j\frac{\cos(\theta + j\alpha)}{\rho a} p\Delta h_2\right] = 0 \tag{2.6}$$

$$W_i(\psi) = \begin{cases} W_*(\psi), & \psi(L_i) < \psi \leqslant \psi(C_i) \\ W_*(\psi) + Z_{L_i}, & \psi(F_i) < \psi \leqslant \psi(L_i) \\ W_*(\psi) + Z_{L_i} + Z_{F_i}, & 0 \leqslant \psi \leqslant \psi(F_i) \end{cases}$$

$$W_*(\psi) = \int_{\psi}^{\psi(C_i)} \left[\frac{v}{u}\frac{dh_1}{d\psi} - (p-p_\infty)\frac{dh_2}{d\psi}\right]d\psi + \left[(p-p_\infty)h_2 - \frac{v}{u}h_1\right]_{C_i}, \quad Z = (p-p_\infty)\Delta h_2 - \frac{v}{u}\Delta h_1$$

In Eq. (2.6) the first term is calculated at the point m on the shock wave, the second is the integral along the streamline mn, the third is calculated at the point n on the closing characteristic, and the fourth at the points S_j—points of intersection of the streamline mn with the lines of discontinuity of the multipliers. If the discontinuity propagates along the characteristic of the first family, we have $j = 1$, otherwise $j = -1$.

The Weierstrass–Erdman conditions at the points of the sharp bends of the airfoil D_1, D_2, E_1 and E_2

$$H^{(1)}(\sigma_0,\mu_0)_{D_1} = 0, \qquad H^{(2)}(\lambda,\mu_0)_{D_1+} = H^{(2)}(\lambda,\mu_0)_{D_1-}$$

$$H^{(1)}(-\sigma_0,-\mu_0)_{D_2} = 0, \quad H^{(2)}(\lambda,-\mu_0)_{D_2+} = H^{(2)}(\lambda,-\mu_0)_{D_2-}$$

$$H^{(1)}(\sigma_0,\mu_0)_{E_1} + H^{(1)}(-\sigma_0,-\mu_0)_{E_2} = 0$$

$$H^{(2)}(\lambda,\mu_0)_{E_1-} - H^{(2)}(0,\mu_0)_{E_1+} = H^{(2)}(\lambda,-\mu_0)_{E_2-} - H^{(2)}(0,-\mu_0)_{E_2+}$$

close the system of necessary conditions, where

$$H^{(1)}(\sigma_0,\mu_0) = \left[p(\sigma_0+\mu_0 x) + h_1\frac{v}{u}\right]_+ - \left[p(\sigma_0+\mu_0 x) + h_1\frac{v}{u}\right]_- + \int_{\omega_+}^{\omega_-}\left[h_1\frac{d}{d\omega}\left(\frac{v}{u}\right) - h_2\frac{dp}{d\omega}\right]d\omega$$

$$H^{(2)}(\lambda,\mu_0) = p(1-\mu_0 y) + \lambda + h_1$$

and ω is the angle of inclination between the characteristic pencil at the point of the sharp bend of the profile and the x axis (in the x, ψ plane); the minus sign indicates the limiting value of a quantity when the point of the sharp bend is approached from the left, while a plus sign indicates the limiting value when approaching it from the right.

From these necessary conditions, relations are obtained which will be used in the numerical algorithm. First, along the optimal closing characteristic C_iB the Lagrange multipliers must be determined from the formulae

$$h_1 = c_3 g, \quad h_2 = c_3 / g \tag{2.7}$$

while the values of the discontinuities of these multipliers along the discontinuity lines (on the characteristics) are related by the equations

$$\Delta h_1 = \mp c_4 g, \quad \Delta h_2 = c_4 / g \tag{2.8}$$

where c_3 and c_4 are constants [4–7].

Second, the equations

$$\begin{aligned} h_1(x,\psi) &= \int_{x_n}^{x} \frac{dp}{dt}(t,\psi)[(-1)^{i-1}\mu_0 y(t,\psi) - 1]dt + h_1(x_n,\psi) \\ h_2(x,\psi) &= \frac{v}{u} + \left[\sigma_0 + \mu_0\left(x + y\frac{v}{u}\right)\right](-1)^i \end{aligned} \tag{2.9}$$

are a solution of system (2.1) with boundary conditions (2.3) in the region P_iE_iB ($i = 1, 2$), which can be verified by direct substitution. Consequently, on the optimal characteristic P_iB the following relation necessarily holds

$$\left\{\frac{v}{u} + \left[\sigma_0 + \mu_0\left(x + y\frac{v}{u}\right)\right](-1)^i\right\}g = c_3 \tag{2.10}$$

The use of the compatibility condition for the flow parameters along the closing characteristic and of Eq. (2.10) enables us to obtain one other relation

$$d\left\{p + g^2\left[\frac{v}{u} + \left(\sigma_0 + \mu_0\left(x + y\frac{v}{u}\right)\right)(-1)^i\right]\right\} + g^2 d\left[\mu_0\left(x + y\frac{v}{u}\right)(-1)^i\right] = 0 \tag{2.11}$$

If the amount of the airfoil is not specified (i.e. $\mu_0 = 0$), differential relation (2.11) becomes the final relation

$$p + g^2\left[\frac{v}{u} + (-1)^i\sigma_0\right] = c_5$$

(c_5 is a constant).

We draw attention to the fact that the system of necessary conditions for an extremum obtained above is not the sum of the necessary conditions for optimizing the upper and lower parts of the airfoil individually. The relation between the upper and lower contours is obtained by using the Weierstrass–Erdman conditions at the points E_1 and E_2.

An analysis of the necessary conditions for an extremum shows that, as in [6], one must introduce an infinite number of points of discontinuity of the airfoil contour along the section AD_i, bunching to the point A. However, it was pointed out in [6] that, in view of the accuracy of the calculations, it was sufficient to confine ourselves to considering discontinuities solely at the points D_i. This will also be used later.

3. DESCRIPTION OF THE NUMERICAL ALGORITHM

To determine the optimum airfoil numerically we propose an iterative process which is essentially a development of the numerical algorithm described earlier in [6].

The abscissa of the point M_1 is specified. The section $[x(A), x(M_1)]$ is divided into n equal parts, we make the number σ_k, the tangent of the slope of the shock wave to the x axis, correspond to each of the points x_k ($k = 1, \ldots n$) obtained, and we set up the shock wave AM_1. From the known free stream and the shock wave we use the method of characteristics [8] to calculate the flow in the region of influence of the line AM_1 and we distinguish the streamline AD_1. We then choose certain negative angles of the sharp bend of the airfoil $\Delta\theta_1$ and $\Delta\theta_2$, by means of which we calculate the rarefaction flow in the region $M_1B_1N_1C_1$ and we set up the section of the shock wave M_1C_1. For an arbitrarily specified value of the stream function at the point G_1 the section $[\psi(G_1), \psi(N_1)]$ is divided into n_1 equal parts, and at the points of division we arbitrarily choose values of the quantity v/u. This information is sufficient to determine the characteristic N_1G_1 and the gas-dynamic quantities on it. Using the method of characteristics we solve Goursat's problem with data on N_1B_1 and N_1G_1 and we distinguish the streamline D_1E_1. By specifying the negative value of the angle of the sharp bend of the airfoil $\Delta\theta_3$ at the point E_1, we calculate the flow in the rarefaction pencil $G_1E_1P_1$. By using relation (2.10) we can determine the optimal characteristic P_1B up to values of the stream function equal to $\psi(A)$ and then separate the streamline E_1B. Hence, the upper part of the airfoil AD_1E_1B and the flow parameters in the region R_1 are known.

We can similarly construct the lower part of the airfoil AD_2E_2B, i.e. specify the abscissa of the point M_2, the shock wave AM_2, the angles of the sharp bend of the airfoil $\Delta\theta_4$, $\Delta\theta_5$, $\Delta\theta_6$, the value of the stream function ψ at the point G_2, and the distribution of the quantity v/u along the line N_2G_2.

The airfoil obtained, generally speaking, does not satisfy the closure condition (the coordinates of the point B along the sections AD_1E_1B and AD_2E_2B are not the same) and the condition for the section E_1E_2 to be orthogonal to the chord AB. One can attempt to satisfy these conditions by choosing the values of $x(M_2)$, $\psi(G_2)$ and $\Delta\theta_6$.

The next step consists of solving the conjugate problem. On the characteristic G_1B the Lagrange multipliers h_1 and h_2 are found from (2.7), while on the characteristic P_1E_1 they are found from (2.9). Then, in the region $G_1E_1P_1$ they are obtained by solving Goursat's problem. At the point E_1 we introduce a discontinuity in the Lagrange multipliers of such a value that on approaching the point E_1 from the left along the airfoil the boundary condition (2.3) for h_2 is satisfied. Using the known distributions of the Lagrange multipliers along B_1E_1 and E_1G_1, the method of characteristics is used to determine h_1 and h_2 in the region $N_1D_1E_1G_1$. On the characteristic N_1G_1 we verify that condition (2.4) is satisfied. At the point G_1 it holds by virtue of the choice of the multipliers h_1 and h_2. The n_1 arbitrariness in choosing the value of v/u on the characteristic N_1G_1 enables it to be satisfied at the remaining n_1 points. In the region $C_1D_1N_1$ the multipliers h_1 and h_2 are determined in the same way as in the region $G_1E_1P_1$. We introduce a discontinuity of the Lagrange multipliers along the characteristic C_1D_1 such that condition (2.6) at the point C_1 holds. The multipliers h_1 and h_2 are determined in terms of their values on the lines AB_1 and D_1M_1. The initial value of the discontinuity of the multipliers along the characteristic F_1B_1 is chosen from the same considerations as at the point E_1.

For the region R_2, the values of h_1 and h_2 are found similarly. After solving the conjugate problem one verifies that the Weierstrass–Erdman conditions are satisfied. Generally speaking, they are not satisfied. One can endeavour to satisfy the Weierstrass–Erdman conditions by choosing the values of $\Delta\theta_1$, $\Delta\theta_2$, $\Delta\theta_4$, $\Delta\theta_5$, $\psi(G_1)$ and $\Delta\theta_3$.

Finally, from the known field of gas-dynamic quantities and the field of the Lagrange multipliers one can calculate the function $E_*^i(\psi)$ for each of the individual streamlines. The shock wave is corrected until $E_*^i(\psi)$ becomes equal to zero. All the calculations must then be carried out again, if necessary.

The airfoil obtained by this procedure is the required optimum airfoil if the coordinates of the point B obtained, the length of the section E_1E_2, the lift force and the moment are equal to the specified values. If this does not occur, one can attempt to achieve it by choosing the values of $x(M_1)$, $\sigma[x(M_1)]$, $\sigma[x(M_2)]$, σ_0, μ_0. When solving specific problems the iterative procedure described can be simplified by using the particular features of the problem.

4. RESULTS OF CALCULATIONS

The iterative algorithm described above was used to solve a number of particular problems. Before presenting the results we will consider the nature of the reflection of the discontinuity of the multipliers h_1 and h_2 from the shock wave and the various solution schemes that follow from it. The value of the reflected discontinuity is found from the condition of continuity of the function $E_*^i(\psi)$ as one approaches the reflection point along the shock wave from the top and from the bottom. We will mean by the sign of the discontinuity the sign of the quantity Δh_2—the difference between the values of h_2 from the left and from the right of the discontinuity. The signs of the discontinuity arriving at the shock wave and of the discontinuity reflected from it may be either the same or different. This depends on the free-stream velocity and the intensity of the shock wave at this point. All the points of the shock wave in the (free-stream velocity—shock-wave intensity) plane will consequently be split into points belonging to regions I and III (the sign of the reflected discontinuity is the same as the sign of the incident discontinuity) and region II (these signs are opposite) [6]. By equating h_1 and h_2 to zero on the right of the characteristic C_iB, we obtain that Eqs (2.7) define the value of the discontinuity of these multipliers along the characteristic C_iB. In this case the discontinuity of the multipliers which propagates along the characteristic C_iD_i is a reflection from the shock wave at the point C_i of the discontinuity propagating along the characteristic C_iB.

Suppose we consider the problem of finding the symmetrical optimum airfoil, placed at zero angle of attack. In this case the point B lies on the x axis, and σ_0 and μ_0 are equal to zero. The value of v/u at the point B is negative, and hence the discontinuity of the Lagrange multipliers along the characteristic C_iB will also be negative. We choose the shock wave so that the parameters corresponding to it belong to region II. Then a discontinuity of positive sign will propagate along the characteristic C_iD_i, and the introduction of a negative angle of the sharp bend in the airfoil at the point D_i enables the appropriate Weierstrass–Erdman conditions to be satisfied. The discontinuity in the multipliers, introduced at the point E_i, will be positive, and on reflection from the shock wave at the point L_i it should change sign. Consequently, the scheme for the solution should be such that an internal shock wave arrives at the point L_i (or a centred compression wave with focus at the point L_i). This considerably complicates the formulation of the problem and the calculations. If we do not introduce reflection of the discontinuity of the Lagrange multipliers from the shock wave at the point L_i the function $E_*^i(\psi)$ will suffer a discontinuity at the point L_i. By assuming this discontinuity to be small (this is in fact the case for chosen parameters of the problem), when carrying out the calculations we will require the function to vanish at those individual points where the shock wave is specified. This corresponds to "smearing" of the local discontinuity at the grid step. In the examples considered the reflected discontinuity at the point D_i had a positive sign. Its value can be neglected to within the accuracy of these calculations. The value of the negative angle of the sharp bend of the airfoil at the point D_i, as can be seen from the results presented below, is also small. The "smeared-out" sharp bend of the airfoil corresponds to the "smeared-out" sharp bend of the shock wave. The parts of the airfoil between the sharp bends, with the exception of the neighbourhood of the "smeared-out" discontinuity, are close to rectilinear.

Below we present the parameters of the optimum symmetric airfoil at zero angle of attack (here $x(B)/l = y(B)/l = 0$)

w_∞	1,4	1,4	1,4	1,5	1,6	1,7
$x(D_1)/l$	0,0349	0,0798	0.1203	0,1392	0,0630	0,0873
$y(D_1)/l$	0,0012	0,0057	0,0134	0,0214	0,0047	0,0102
$x(E_1)/l$	0.5259	0,5553	0.5891	0.6317	0,5531	0,5941
$y(E_1)/l$	0,0193	0,0433	0,0668	0.0988	0,0408	0,0669
T/l, %	3.9	8,7	13,4	19,8	8,2	13,4
C_X	0,0025	0,0128	0,0295	0.0533	0,0082	0,0181
$\Delta\theta_1+\Delta\theta_2$	0,0002	0,0012	0,0023	0,0045	0,0007	0,0009
$\Delta\theta_3$	0,0696	0,1742	0,2742	0.4311	0,1663	0,2545

In Fig. 2 we show the upper half of the optimum airfoil obtained with a thickening of T/l = 19.8%. The constants C_4, which occur in relations (2.8) and which characterize the discontinuity of the multipliers, have the following values for this profile: $c_{E_1L_1} = 0.4744$, $c_{C_1D_1} = 0.0079$, $c_{D_1F_1} = 0.0001$, $c_{L_1Q_1} = -0.0149$.

In the case of asymmetrical airfoils at non-zero angle of attack, when carrying out the calculations the initial shock wave was chosen to be in the neighbourhood of the boundary of regions II and III, which enabled us to neglect the value of the discontinuity of the multipliers at the point C_i and not to introduce a sharp bend in the airfoil at the points D_i. The factors σ_0 and μ_0 were chosen to be non-zero here.

The following are the parameters of the optimum airfoils obtained:

(a) airfoil movements not specified, $\mu_0 = 0$

w_∞	1.828	1.828	1.816	1.828
γ	−0.0029	−0.0233	0.0366	0.0191
$x(E_1)/l$	0.5136	0.5976	0,6201	0.5253
$y(E_1)/l$	0.0025	0,0272	0.0457	0.0036
$x(E_2)/l$	0.5136	0.5988	0.6157	0,5241
$y(E_2)/l$	0,0124	0.0359	0,0502	0,0133
$x(B)/l$	1.0000	0.9997	0.9993	0.9998
$y(B)/l$	0.0025	0.0235	−0.0368	−0.0193
T/l, %	1.5	6.3	9.6	1.7
C_X	0.0004	0,0044	0.0098	0,0007
C_Y	−0,0026	−0,0148	0.0220	0,0155
C_M	−0.0046	−0,0137	0,0166	0.0087
σ_0	−0.3	−0.3	0.15	0,35
$\Delta\theta_3$	0.0181	0,5345	0.2917	0,0525
$\Delta\theta_6$	0.0502	0.2064	0.1184	0.0128

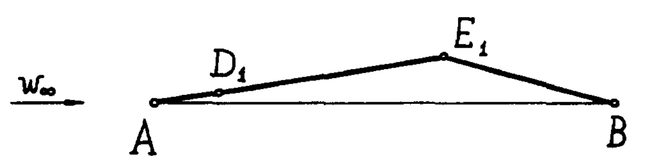

Fig. 2.

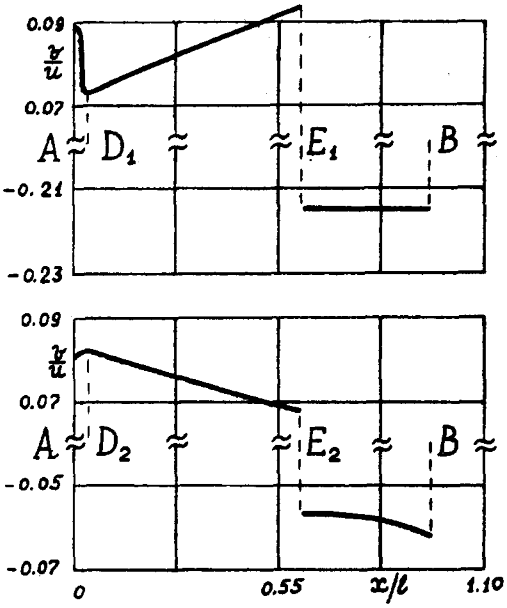

Fig. 3.

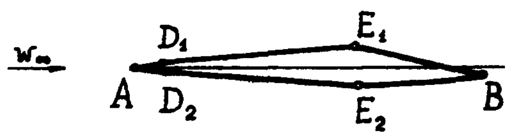

Fig. 4.

(b) airfoil movements specified, $\mu_0 \neq 0$, and $\sigma_0 = 0.15$

w_∞	1,816	1,816	1,816	1,816	1,816	1,816
γ	0,0335	0,0326	0,0307	0,0293	0,0279	0,0264
$x(E_1)/l$	0,6230	0,6237	0,6189	0,6273	0,6287	0,6302
$y(E_1)/l$	0,0482	0,0482	0,0490	0,0508	0,0520	0,0521
$x(E_2)/l$	0,6196	0,6203	0,6167	0,6250	0,6265	0,6279
$y(E_2)/l$	0,0493	0,0494	0,0479	0,0485	0,0486	0,0476
$x(B)/l$	0,9994	0,9995	0,9996	0,9996	0,9996	0,9997
$y(B)/l$	−0,0336	−0,0325	−0,0301	−0,0293	−0,0283	−0,0261
T/l, %	9,8	9,8	9,9	9,9	10,0	10,0
C_X	0,0095	0,0095	0,0098	0,0099	0,0099	0,0100
C_Y	0,0194	0,0187	0,0169	0,0161	0,0147	0,0134
C_M	0,0149	0,0145	0,0135	0,0131	0,0124	0,0117
μ_0	0,2	0,3	0,4	0,5	0,6	0,7
$\Delta\theta_3$	0,3273	0,2988	0,3016	0,3038	0,3064	0,3086
$\Delta\theta_6$	0,1220	0,1204	0,1231	0,1236	0,1245	0,1254

If the moment of the airfoil is not specified, the parts of the profile between the sharp bends are close to rectilinear. If the moment of the airfoil is specified, the parts between the sharp bends become bent. The bending is particularly pronounced along the part D_iE_i. In Fig. 3 we show the distribution of v/u along the length of the projection of the chord onto the free stream direction for the airfoil, obtained for $\mu_0 = 0.7$, and in Fig. 4 for the airfoil itself.

In the above calculations the gas was assumed to be perfect with an adiabatic index $\varkappa = 1.4$. The function $E_*^i(\psi)$ at points on the shock wave in the calculated examples had a value of the order of 10^{-4}, which corresponds to the accuracy of the numerical method of characteristics for the chosen step. All the linear dimensions are normalized to the length of the chord l and $C_X = X/(l\rho_\infty w_\infty^2)$, $C_Y = Y(l\rho_\infty w_\infty^2)$, $C_M = M/(l^2\rho_\infty w_\infty^2)$.

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